AcmeCorp tundish pour - quantitative prediction in design

Dr. Richard D. Smith

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Planning - the guidance offered by physics

The most elementary relationship / equation for analyzing fluid flow is the Bernoulli equation. This is because it is a simple conservation-of-energy expression.

The Bernoulli equation states that

$$\rho gh + \frac{1}{2}\rho v^2 + P = \text{constant}$$

where ρ is fluid density, g is gravitational acceleration and is considered constant, h is the height of the reference point above a datum, v is the velocity of the fluid and P is the pressure of the fluid.

Each of the three terms is a part of the energy which the fluid possesses and the sum of the three must remain a constant. The first term, "ρgh", is the potential energy of the fluid due to altitude in a gravitational field. The second term is the kinetic energy of the fluid due to its velocity. The third term is the energy due to pressure.

In the case of the tundish the obvious choice is to make "condition 1" the conditions at the surface of the melt in the tundish while "condition 2" are the conditions as the melt flow leaves the nozzle at the base of the tundish. We can then state

$$\rho_1 gh_1 + \frac{1}{2}\rho_1 v_1^2 + P_1 = \rho_2 gh_2 + \frac{1}{2}\rho_2 v_2^2 + P_2$$

where the variables take on the following meaning: h_1, v_1 and P_1 are the height, velocity and pressure of the melt at the melt surface in the tundish, while h_2, v_2 and P_2 are the corresponding values at the base of the nozzle.

For the case of a tundish and with the reference points selected, the Bernoulli equation can be much-reduced.

The density of the melt, ρ, is a constant, as the fluid is a liquid and therefore incompressible. Also, P_1 and P_2 are both atmospheric pressure and can be canceled from the equation, so that

$$\rho gh_1 + \frac{1}{2}\rho v_1^2 = \rho gh_2 + \frac{1}{2}\rho v_2^2$$

The remaining terms all share ρ so this can be canceled and with some reorganization we get

$$g\left(h_1 - h_2\right) = \frac{1}{2}\left(v_2^2 - v_1^2\right)$$

A final reduction of the Bernoulli equation in the case of a tundish is to assume that the velocity at the melt surface is negligible. This would be a true if the surface area of the tundish trough as seen in plan is much greater than the area of the exit nozzle of the tundish. This is indeed a very reasonable approximation.

Renaming v_2 to v_n to indicate velocity out of the tundish nozzle and $(h_1 - h_2)$ to h_m signifying the height of the metal in the tundish from the base of the nozzle to the melt surface in the tundish, we can finally derive

$$gh_m = \frac{1}{2}v_n^2$$

This says that the velocity of the fluid – any fluid – out of the nozzle is a function of gravity and the depth of the melt from the melt surface in the tundish to the exit of the nozzle.

The velocity of the melt leaving the nozzle is very useful to us. If the area of the nozzle is known, which could be referred to as a_n, the volume flow of melt per unit time through the tundish nozzle, which we could name $\dot{q}_n$, is

$$\dot{q}_n = v_n a_n$$

Alternatively, $\dot{q}_n$ can be calculated as the dependent variable of two predefined features of the casting procedure. One of these is the volume of the ingot molds, or more precisely stated, the volume of metal which the ingot molds contain when they are filled to the selected level near their top. Secondly, we can visualize the elapse of time we wish to pass in the filling of each ingot mold. From this desired $\dot{q}_n$, the correct value of a_n can be calculated and from this the correct diameter of the nozzle becomes known.

Recapitulating on the guidance which has been obtained from physics and simple mathematics, we find that for the entire tundish pour there is one crucial dimension which is the diameter of the tundish nozzle. To obtain the correct flow rate and the correct depth of metal in the tundish it is found that both these features depend on the tundish nozzle diameter.

Combining the guidance from physics with the hardware of the "AcmeCorp pour"

The "AcmeCorp molds" are suitably filled by a volume of 0.0011m^3 of metal (as the overall dimensions of the ingot in inches are about 11 1/2in long by 2 1/4in mean width by 2 1/2in tall).

An aim fill time of 8 seconds for each ingot mold was decided upon. This was on the basis that

- this would give a steady rise of the metal in the molds without vigorous turbulence, on the basis of past general experience
- the manual loading of ingot molds ready to be filled would be manageable at this rate
- as the flow could not be stopped, the potential amount of spatter as the ingot molds are moved to present the next empty mold, an operation of about half a second, becomes a small part of the total metal poured (spatter in only 1/2 second out of 8 seconds)
- the furnace will be emptied of metal and all the ingots cast in a reasonable length of time which is not excessive

From the volume of metal which the mold holds and the duration of the pour, we calculate that

$$\dot{q}_n = 0.0001375\text{m}^3\text{s}^{-1}$$

Utilizing previously derived expressions:

For nozzle velocity:

$$gh_m = \frac{1}{2}v_n^2 \iff v_n = \sqrt{2gh_m}$$

The "head" of metal from the outlet of the tundish nozzle to the metal level in the tundish, h_m, was chosen to be about 8 inches, for which is conveniently expressed as 0.2m .

A convenient approximation for a gravitational acceleration of 9.8ms^{-2} is to take a value for g of 10, so that

$$v_n = 2\text{ms}^{-1}$$

This now leads us to calculating the nozzle area:

$$\dot{q}_n = v_n a_n \iff a_n = \frac{\dot{q}_n}{v_n}$$

so

$$a_n = 6.875 \times 10^{-5}\ \text{m}^2$$

and this leads us finally to the diameter of the tundish nozzle via the equation for the relation between the diameter and area of a circle

$$a = \frac{\pi d^2}{4} \iff d = \sqrt{\frac{4a}{\pi}}$$

so that

$$d_n = 0.0094\text{m}$$

That is, we finally arrive at a calculated value that the nozzle of the tundish should have a diameter of 9.4mm which is close to 3/8 inch (9.52mm).

Given the latitude inherent in the design criteria of the fill time for the ingot molds and the height of metal maintained in the tundish, a tundish nozzle diameter of 3/8ths of an inch would be fine.

This proved to be the case in practice. The simple energy-conservation Bernoulli equation was applied and there was no obvious deviation between the expected outcome and the observed outcome, implying that more sophisticated considerations related to fluid flow thought of as "drag" were not significant in this application.

It is again re-iterated that for a tundish pouring operation, the entire design revolves around the tundish nozzle diameter. The correct specification of tundish nozzle diameter enables both the flow of metal into the ingot molds at the desired pour rate and the chosen fill height of liquid metal in the tundish to be maintained.